Localization Tutorial

Phoenix Project

November 9, 2015

Funded by the
European Union

Outline

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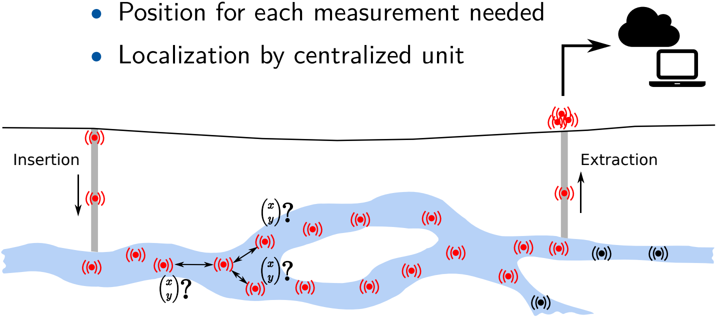
Localization Process in Phoenix

Project goal

- Explore underground systems with sensors
- Sensor measurements enable e.g. temperature or pressure profiles
- Sensor positioning needed for measurement mapping
  - Position for each measurement needed
  - Localization by centralized unit



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Localization techniques

- **Anchor-based:** A subset of the nodes have a priori known positions. Nodes with unknown position use positioning information from anchors to determine their absolute position
- **Anchor-free:** No a priori knowledge on the positions of any node. Only relative coordinates can be obtained

- **Range-based:** Relative distances to other nodes are measured
- **Angle-based:** Angle information to other nodes is measured
- **Range-free:** Only connectivity information available

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Range-based Localization

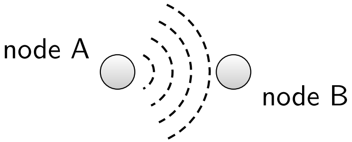


Figure: RSS measurement

System model:

$$P_r(d) = P_0 - 10\gamma \log_{10} d + S$$

$$\mathtt{Var}(\hat{d}) \geqslant \left(\frac{\ln 10 \sigma_s}{10\gamma} d\right)^2$$

S: shadowing with variance σ_s^2
γ: path-loss exponent

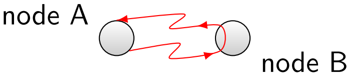


Figure: Two-way TOA ranging

System model:

$$r(t) = \sqrt{E} p(t - \tau) + n(t)$$

$$\mathtt{Var}(\hat{d}) \geqslant \frac{c^2}{8\pi^2 \beta^2 \, \mathtt{SNR}}$$

$\mathtt{SNR} \triangleq E/N_0$
β: effective bandwidth
$\beta^2 = \int_{\mathbb{R}} f^2 |P(f)|^2 \, df / \int_{\mathbb{R}} |P(f)|^2 \, df$

Range-based Localization

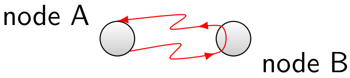


Figure: Two-way TOA ranging

- Bidirectional communication required
- Data stored by each sensor:
    - ID of node B
    - Time stamp
    - ToF or distance to node B (can be used for online sensor adaptation)

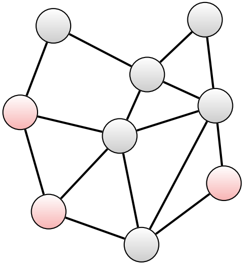
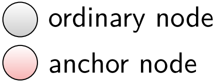


Figure: Example Network



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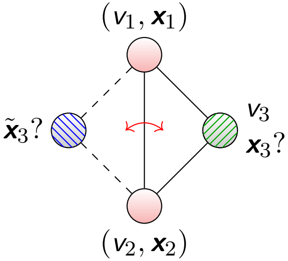
Range-based Localization



Localization problems due to ambiguities

- Consider simplified, noise-less case in 2D:
  - Two fixed nodes v_1, v_2: known position
  - Localize v_3's position
  - same measurements $\rightarrow$ different solutions

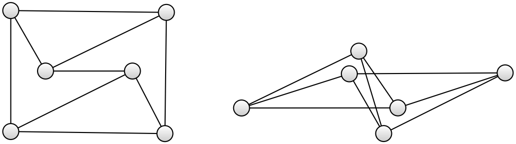


Figure: 3-connected graph with flex-ambiguity

- Realistic, noisy case:
  - Even more ambiguities present due to noise variance

Range-based Localization

Impact of noisy range measurements

- Three anchor (or already localized) nodes
- Fourth sensors to be localized
- Simplified case: noise only on measurements of v_3
  - By range measurement d_{34}, $\tilde{d}_{34}$: two possible solutions: $x, \tilde{x}$

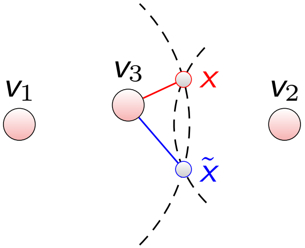


Figure: Network with noisy range measurements

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Range-based Localization

Complexity of Localization in noise-less case

The localization of sensors by means of range measurements is $\mathcal{NP}$-hard and it's of combinatorial nature (Complexity $\overset{a.s.}{\sim}$ exp(#sensors)).

- Special cases
  - Trilateration (Quadrilateration) graphs: polynomial (or even linear) in # sensors
  - There exists an ordering for $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ in $\mathbb{R}^d$ such that the sensors can be localized by $|\mathcal{V}| - d$ consecutive trilaterations

Range-based Localization

- Unique localizability constraints for noisy case unknown
- For noise-less case in d dimensions
  - at least $d + 1$ anchors in *general* position
  - corresponding graph must be *globally rigid* and $(d + 1)$-connected
    - k-connectivity: removing any $k - 1$ vertices doesn't render $\mathcal{G}$ unconnected
- Noise-less case in 2D:
  - Graph $\mathcal{G}$ 6-connected $\Rightarrow$ global rigidity
  - Graph $\mathcal{G}(R)$ 2-connected $\Rightarrow$ $\mathcal{G}(2R)$ globally rigid

    R: sensor communication range
- Noise-less case in 3D:
  - Graph $\mathcal{G}(R)$ 2-connected $\Rightarrow$ $\mathcal{G}(3R)$ globally rigid

$\Rightarrow$ High connectivity essential for sensor localizability

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Summary

Steps obtain sensor positions

- Collect timing or distance measurements from all agents
- Perform joint localization
  - Estimate agent positions at each measurement instance

Unique localization requirements in noise-less 3D case

- Minimal requirements: at least 4-connectivity
- No constructive criteria known
- Conjecture: 12-connectivity $\Rightarrow$ global rigidity

References

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